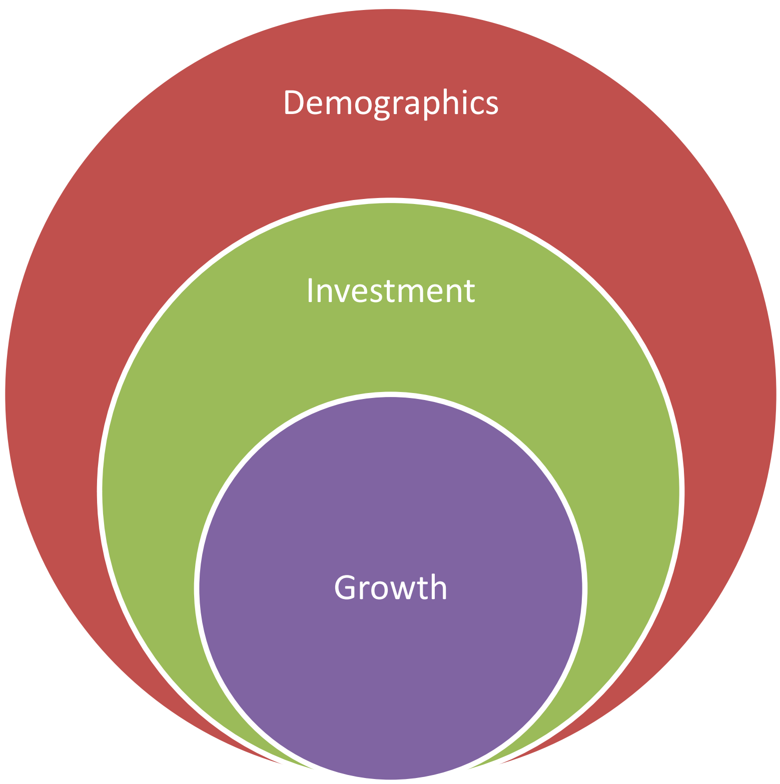
CASE STUDY: THE *DIG* MODEL

A SYSTEM DYNAMICS ANALYSIS OF DEMOGRAPHICS, INVESTMENT DECISIONS, AND COMPANY GROWTH



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INTRODUCTION

In this paper, a model is developed relating company growth and contraction to demographic changes. Investment decisions connect the two. The model is dubbed "DIG" as an acronym for Demographics-Investment-Growth. The system dynamics technique is used to construct this model. System dynamics is a powerful analytical technique that may be used to analyze non-steady-state systems containing multiple components and subsystems. A standard system dynamics model contains a set of "stocks" or "reservoirs" that store some material. The stocks are connected by a set of "flows" that allow the material to move from store to store. These flows can be modeled as a set of differential equation. The system dynamics analysis technique has been most broadly adopted in the environmental and ecological fields. Many environmental problems such as the modeling of deer population, or pollutant flows, or energy generation lend themselves well to the system dynamics technique (Ford, 1999). System dynamics has not found much of an acceptance or popularity in the economics field. Nevertheless, it has obtained some entryway into the discipline (White, May 1992).

One particularly powerful modeling application is Simgua[1]. Simgua is an extensible and easy to use tool with which we have implemented the DIG model. In the rest of this paper, we will first carry out a literature review of relevant research, then develop the DIG model in detail, and finally test and analyze the results of the model.

LITERATURE REVIEW

Most of the research available that is relevant to this subject deals with demographic changes and their effect on investment decisions and asset prices. The impetus for much of this work comes from the aging of the baby boom generation. Many researchers speculate, and their models indicate, that as baby boomers age and enter retirement they will begin to sell off their assets thereby pushing asset prices down and potentially causing economic turmoil. The other theory is that the market, being efficient, will have already taken into account the expected retirement of the baby boomers and that asset prices should already reflect the predicted selloff. One paper analyzing the effects of baby boomers on asset prices develops a model predicting asset prices given demographic changes (Brooks, April 2000). In this model, the population is split into four groups: children, young workers, old workers, and retirees. The different groups possess differing access to

[1] http://www.simgua.com/ - Developed by Your Lingua software.

assets. In the model, individuals make investment decisions based on their current position in life. For instance:

> *"YOUNG WORKERS ANTICIPATE RECEIVING WAGE INCOME IN OLD WORKING-AGE, SO THAT NEXT PERIOD CONSUMPTION DOES NOT DEPEND ON SAVINGS ALONE. IN ADDITION, SINCE THE RETURN ON CAPITAL IS POSITIVELY BUT IMPERFECTLY CORRELATED WITH WAGE INCOME, HOLDING EQUITY WILL DIVERSIFY THE EFFECTS OF AN ADVERSE TECHNOLOGIC SHOCK. IN CONTRAST, OLD WORKERS' INVESTMENT DECISION REFLECTS THE FACT THAT NEXT PERIOD CONSUMPTION IS OUT OF THE SAVINGS ALONE. AS A RESULT THEY LARGELY ELIMINATE CONSUMPTION RISK BY INVESTING MOSTLY IN THE RISKLESS ASSET."*
>
> *(BROOKS, 2)*

The basic effect of the model's investment decisions is that older workers are more risk adverse than younger workers. The author's conclusion in this paper is that changes in demography can have significant effects on asset prices and that governments should proactively manipulate the price of risk- free assets to minimize disturbances due to demographic effects.

One of the first studies on the effects of demographic structures on asset returns also carries the assumption that individuals become more risk adverse as they age (Bakshi & Zhiwu, 1994). There is empirical evidence for this assumption that extends back half a century (Lampman, 1962). In addition to the empirical evidence, the assumption appears to be a reasonable and rational one to make: as individuals approach retirement and the need to utilize savings, the importance of a steady income stream increases and willingness to bear risk decreases. Nevertheless, it appears that more recent empirical data and the accounting for a wide range of factors that may possess more explanatory power sheds some doubt on this assumption (Poterba, Demographic structures and asset returns, 2001) (Poterba, Population aging and the financial markets, August 2004).

For instance, while it appears that stock ownership steadily rises with age before declining in retirement this could be just the result of external factors and limited datasets. When the overall behavior of the markets and investor's investment decisions are taken into account there appears to be no age-dependent correlation between equity ownership and age (Ameriks & Zeldes, 2000). Similarly, one expects net worth of individuals to steadily decline with age after a certain point. Data collected from the triennial Survey of Consumer Finances do indicate some decline in net worth but much less than would be expected. There are several reasons that could explain this discrepancy between the expected and the observed (Poterba, Demographic structures and asset returns, 2001):

- A very important factor is the so-called "bequest motive". This is based on the idea that some people save in order to leave a bequest or inheritance of significance to their relatives or favored institutions. If the bequest motive truly does play a large factor, it could cause individuals to accumulate many more assets than they needed to survive and to not spend those funds during retirement.
- Another issue could be a correlation between mortality and wealth. Richer individuals can afford superior healthcare than can poorer individuals. This would mean that impoverished individuals experience a higher mortality rate than richer individuals thereby raising the average net worth of older populations as compared to younger populations.
- A final possible factor is that when a husband or wife dies, they often leave a significant portion of their assets to their spouse. This raises the average net worth of the population as the per capita wealth increases.

In addition to the Brooks model discussed earlier, a number of other demographic and asset pricing models have been developed and are described by Poterba. Several of these models are now briefly touched upon. Peter Yoo developed a model in which it is shown that a baby boom results in an increase in asset prices followed by a decrease in asset prices. The magnitude of this spike is dependent on the variability of the supply of the assets. John Geanakoplos and his colleagues developed a complex model that takes into account bequests, Social Security, accurate age income effects, and more. The result of this model is that the baby boom could cause large spikes in asset prices. Comparing the predictions to observed spikes, however, indicates that the predictions only explain one third to one half of the observed variations (Poterba, Population aging and the financial markets, August 2004).

Clearly the field of using demographics to predict asset prices or economic movements is an immature one. Different sources disagree on such fundamental issues as whether individuals become more risk adverse with age. There is a large variety of competing models that attempt to explain demographic effects. These models take different approaches and yield differing results. No system dynamics models were encountered in this literature review which we believe to be an important lack given system dynamics excellent applicability to the issue.

DIG MODEL FORMULATION

The DIG model is split into three layers. The first layer is the population layer. This layer represents and simulates demographic changes and scenarios for a country, or even the whole world. The population is represented as three separate categories: the first category is the so-called "pre-investor" category, the second category is the "risk loving investor"[2] category, and the third is the "risk adverse investor" category. This first layer directly determines the second layer which represents the distribution of capital between stocks, bonds, and risk-free investments. Risk loving investors obviously invest a greater share of their assets in high-risk, high return stocks, while, conversely, risk adverse investors invest a greater portion of their wealth in bonds and risk-free assets. The final layer is built directly upon the second and models the distribution of small, medium, and large sized companies. This is simulated as a flow between these three classes of companies where small companies can grow to become medium companies, and medium companies can grow to become large companies. Conversely, large companies may experience contractions and become medium companies, and medium companies may experience contractions to become small companies. Small companies may be created and destroyed independently of other companies' growth or contraction.

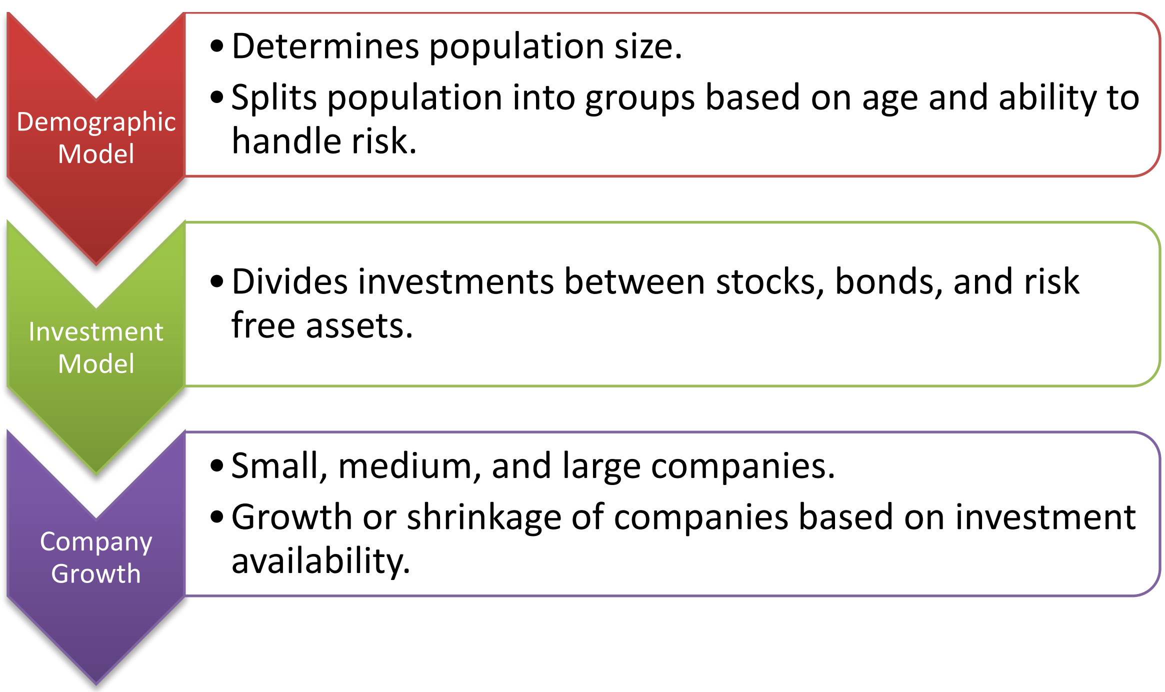

FIGURE 1: DIG MODEL OVERVIEW.

[2] A slight misnomer. These investors will not aggressively pursue risk, but they will invest in more risky asset bundles than the other investor categories. A possibly better categorization of investors would be the following: "unable-to-bear-risk", "strongly-risk-adverse", and "moderately-risk-adverse". This latter naming scheme is unfortunately prohibitively cumbersome.

Let us take a moment to examine each of these three layers in more detail. As discussed earlier, the demographic simulation is the foundation of the entire model. The other two layers are derived directly, or indirectly, from it. There are many different ways we could simulate population changes. Possibly the simplest and most accurate would be to simply feed in demographic data already collected through census surveys or other empirical techniques. This method however, would impose a data collection burden on the experimenter and would also tie his results to a specific population. If the experimenter is not concerned about a specific population, and is more concerned about the broad generalities and repercussions of the model, it could be useful to have a simple population growth model that does not depend on large amounts of demographic data.

The model created for this paper contains basic population growth equations as part of its simulation. As mentioned earlier, the population is divided into three categories: pre-investor, risk loving investor, and risk-averse investor. It is assumed that the pre-investor category represents ages 1 – 18, the risk loving investor represents ages 19 – 50, and the risk adverse investor represents ages 51 – 70. In this simple demographic model, it is assumed individuals die at the age of 70. In our population growth equations we assume that only the risk loving investors are able to reproduce which is a valid approximation given their age range. To calculate the number of births in a time period we multiply the population in the risk loving category by one half to get the number of females and then again by one over the number of time periods in the category so each woman is counted only once. The resultant is then multiplied by a fertility rate coefficient which is the average number of children per woman. The resulting figure is added to the pre-investor category. Simulating aging can be achieved in a similarly simple manner. Each time period the population in the pre-investor category can be multiplied by the number of time periods in that category and the resulting figure moved from the pre-investor category into the risk loving category. The same procedure can be carried out to simulate aging between the risk loving investor category and the risk adverse investor category. Deaths at the age of 70 can also be modeled in this way. The following is a set of differential equations defining the simple population model outlined here:

$$\frac{dPreInvestorPop}{dt} = \frac{RiskLovingPop}{2} * \frac{1}{32} * FertilityRate - PreInvestorPop * \frac{1}{18}$$

$$\frac{dRiskLovingPop}{dt} = PreInvestorPop * \frac{1}{18} - RiskLovingPop * \frac{1}{32}$$

$$\frac{dRiskAdversePop}{dt} = RiskLovingPop * \frac{1}{32} - RiskAdversePop * \frac{1}{20}$$

EQUATION 1: DEMOGRAPHIC MODEL FORMULATION.

It is important that the experimenter properly calibrate this model for the type of population he is trying to study. Particularly close attention needs to be paid to the fertility rate specified. Initial values for the three categories must also be determined and the age ranges for the categories should be specified. If unreasonable parameter values are chosen, the population could quickly skyrocket or crash into nothing. It is also important to note the simplicity of this model. It was designed to be as simple and easy to understand as possible while still yielding enough data to determine the behavior of the layers built on top of it. It does not account for possible deaths in young population groups, its aging behavior is extremely simple and distorts certain demographic effects, and the behavior of births are similarly extremely simplified. Factors such as immigration or emigration must also be taken into account. In future work it would not be difficult and could be recommended to replace the existing population model with a more accurate, but more complex, model.

INVESTMENT MODEL

We allow for three different types of investments: stocks, bonds, and risk-free investments. Stocks are the riskiest of the investments, but they also offer the highest rate of return. Risk-free investments, on the other hand, are just that – risk-free – but they offer the lowest rate return. Bonds are somewhere in the middle of the two, offering average risk and an average rate of return. Depending on an investor's willingness to tolerate risk he will invest in combinations of these three forms of assets in differing proportions. A risk adverse investor will invest larger sums in safe risk-free assets and relatively safe bonds than a risk loving investor will. As discussed in the research section of this paper, it is reasonable to develop correlations between an individual's age and his willingness to tolerate risk.

To develop our model, we assume that the population in the pre-investor category will not be investing in stocks or bonds. We do presume, however, that this age group will possess some small sum of assets that they will store in a bank account: a risk-free asset. We assume the next oldest population is tolerant of risk. This population has a steady income stream and is primarily saving toward retirement. Retirement is far enough off for these individuals that they can pursue an aggressive investing strategy that accepts large amounts of risk in return for high expected appreciations. The last category – individuals aged 50 and over – are considered to be more strongly risk adverse due to steadily diminishing incomes and the proximity of retirement. Given these three categories we then determine the population's distribution of capital between the three asset classes by assigning an asset distribution to each population in class and then summing the distributions between the three classes together. The following set of equations determines the total asset distribution given population sizes and population preferences:

$$RiskFreeCapital = PreInvestorPop * PreInvestorWealthPerCap$$

$$RiskLovingCapital = RiskLovingPop * RiskLovingWealthPerCap$$

$$RiskAdverseCapital = RiskAdversePop * RiskAdversePopPerCap$$

$$RiskFree = RiskFreeCapital * 1 + RiskLovingCapital * RiskLovingRiskFreeCoef + RiskAdverseCapital * RiskAdverseRiskFreeCoef$$

$$Bonds = RiskLovingCapital * RiskLovingBondsCoef + RiskAdverseCapital * RiskAdverseBondsCoef$$

$$Stocks = RiskLovingCapital * RiskLovingStocksCoef + RiskAdverseCapital * RiskAdverseStocksCoef$$

$$1 = RiskAdverseStocksCoef + RiskAdverseBondsCoef + RiskAdverseRiskFreeCoef$$

$$1 = RiskLovingStocksCoef + RiskLovingBondsCoef + RiskLovingRiskFreeCoef$$

EQUATION 2: ASSET DISTRIBUTION FORMULATION.

As with the population model, we should note the significance of calibration for our asset distribution model. The coefficients for the different investments must be determined through empirical observation or theoretical analysis of the situation. It is also important to note that when we say "distribution" or "allocation" we are not referring to the creation of new assets rather we are referring to individuals willingness to buy or new existing assets in that category. A high allocation to some asset indicates a high willingness to buy and therefore a high price.

COMPANY GROWTH MODEL

The final portion of our simulation is the company growth model. The company growth model sits directly on top of the asset distribution model. We define three separate company sizes as outlined in (Ritter, Silber, & Udell): small companies (less than $10 million in assets), medium-sized companies ($10-$150 million), and large companies (over $150 million). In this model, the fundamental differential between these three classes is their access to various kinds of capital. Since we are primarily concerned with the company growth or contraction we can relate these events to the companies' access to specific forms of capital. When an entrepreneur decides to create a small company his success in raising the capital for that company is primarily dependent on his ability to obtain a bank loan or, in other words, his ability to attract risk-free capital. The entrepreneur will be unable to court the bond or stock markets. Thus, in our model, the creation and destruction of small companies is solely determined by the availability of risk-free capital.

Medium and large companies, on the other hand, have access to the stock and bond markets. It is here that we make a very important distinction that underlies the behavior of this model. When a small company grows to a medium-sized company or a medium-sized company grows to a large company, we assume that this growth is funded through some form of stock offering such as an initial public offering (IPO). Conversely, the ability for a large or a medium-sized company to maintain its operations is dependent on its ability to arrange financing through the bond market. Thus, company growth is determined by the availability of stock capital while the company contraction is determined by the availability of bond capital.

The company growth model as outlined here can be modeled with the following set of equations:

$$RiskFreeDemand = NumSmall$$

$$StockDemand = NumSmall + \alpha_s * NumMedium$$

$$BondDemand = NumMedium + \alpha_b * NumLarge$$

$$\frac{dNumSmall}{dt} = \left[\frac{RiskFree}{\beta_{s1}} - RiskFreeDemand\right] * \beta_{s2} - \text{NumSmall} * \frac{\text{Stocks}}{\text{StockDemand} * \beta_{\text{ms}}} + \text{NumMed} * \frac{\text{BondDemand} * \beta_{\text{mb}}}{\text{Bonds}}$$

$$\frac{dNumMed}{dt} = \text{NumSmall} * \frac{\text{Stocks}}{\text{StockDemand} * \beta_{\text{ms}}} - \text{NumMed} * \frac{\text{BondDemand} * \beta_{\text{mb}}}{\text{Bonds}} - \text{NumMed} * \frac{\text{Stocks}}{\text{StockDemand} * \beta_{\text{ls}}} + \text{NumLarge} * \frac{\text{BondDemand} * \beta_{\text{lb}}}{\text{Bonds}}$$

$$\frac{dNumLarge}{dt} = \text{NumMed} * \frac{\text{Stocks}}{\text{StockDemand} * \beta_{\text{ls}}} - \text{NumLarge} * \frac{\text{BondDemand} * \beta_{\text{lb}}}{\text{Bonds}}$$

EQUATION 3: COMPANY GROWTH FORMULATION.

These equations appear rather complex, but when represented in Simgua they are quite elementary. Additionally the form of equations specified here is not the definite version. Possibly, other equations would better represent company growth and contraction. As with the other two models, accurate calibration is an important part of the company growth model. The different parameters and initial values should be determined empirically or through a theoretical analysis of the situation. The assumptions underlying this sub-model – namely, the different effects of stock and bond assets on company growth and contraction – are the most tenuous of the DIG model. In future research, these assumptions should be examined and either confirmed or modified to more accurately reflect reality.

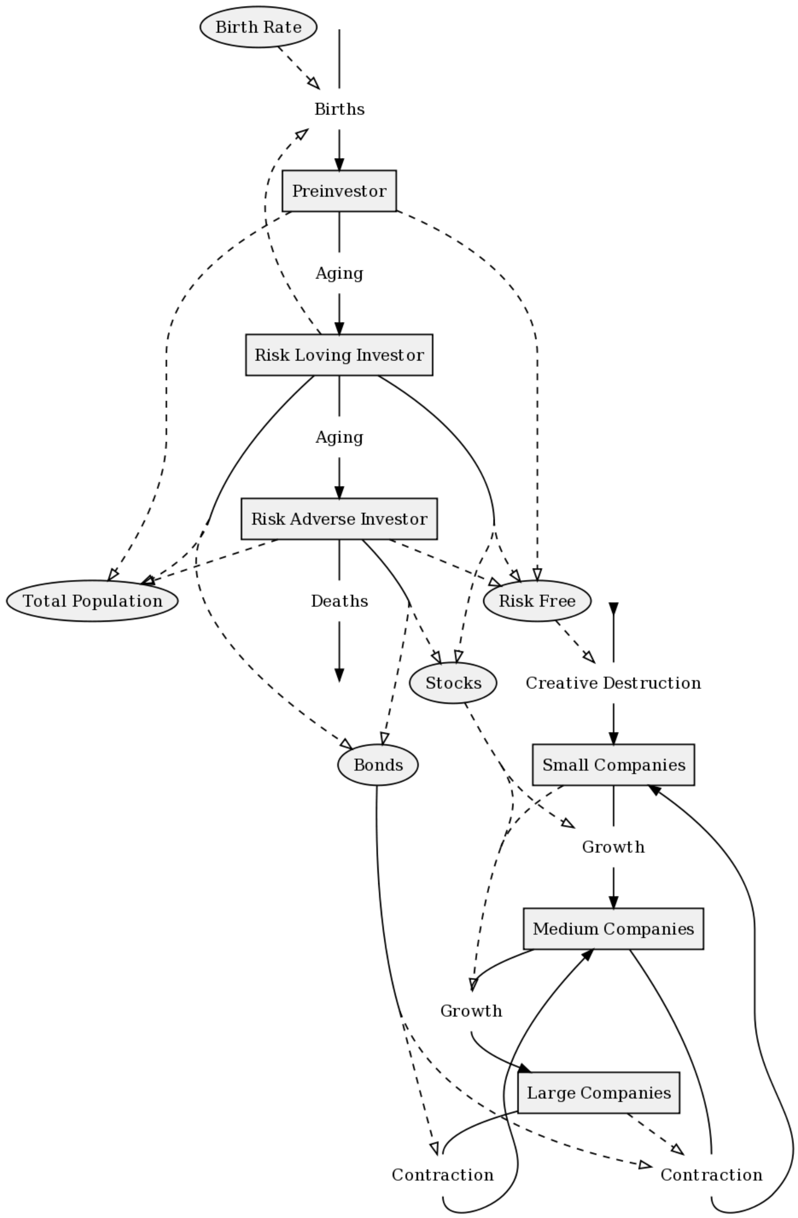

FIGURE 2: GRAPHICAL REPRESENTATION OF THE DIG MODEL.

It is important to note that the DIG model is not a full general equilibrium model and leaves out important issues including the effects of asset prices. A more complete model is outside the scope of the paper, but is suggested for future research.

ANALYSIS RESULTS

The demographic model was calibrated to roughly approximate the United States. The fertility rate was set at 2.09 births per woman (United States Total Fertility Rate). The population of the pre-investor category was estimated to be 80 million, the risk loving investor category was estimated to be 143 million, and the risk adverse investor category was estimated to be 60 million (Bureau, International Data Base (IDB) - Pyramids). Using this data we ran the demographic portion of the DIG model generating the following figure for a 150 year time span:

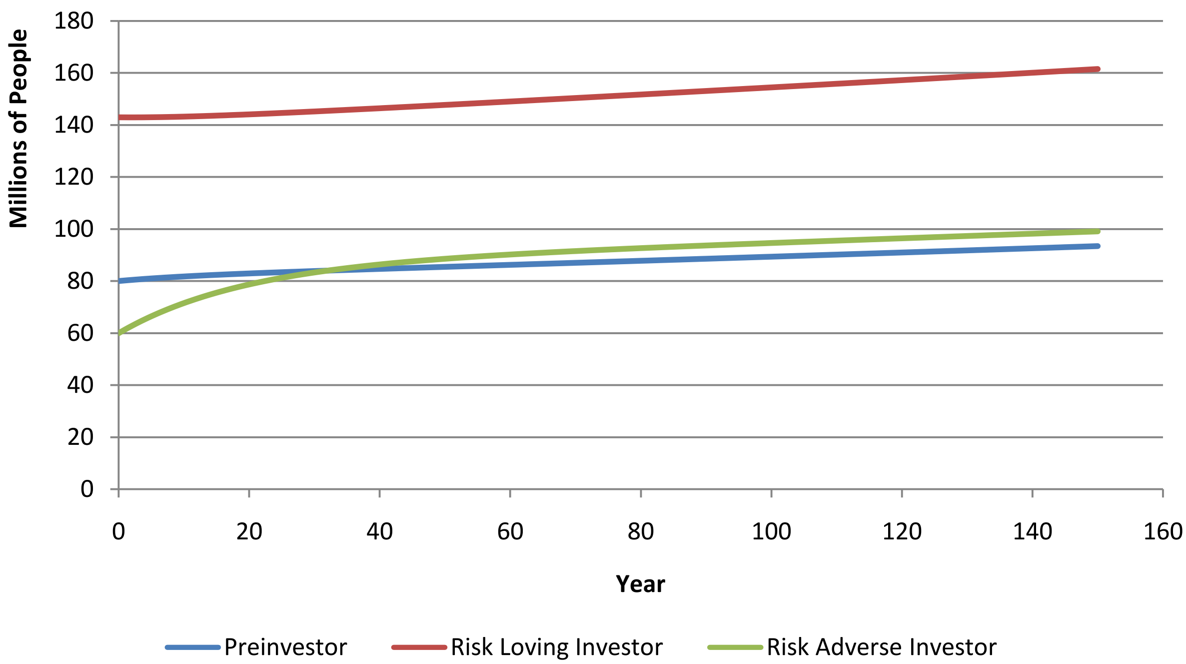

FIGURE 3: DEMOGRAPHIC CHANGES FOR 150 YEARS GIVEN DEMOGRAPHIC MODEL AND PARAMATERS.

It is possible to estimate the per-capita wealth of the different age categories using the Survey of Consumer Finances that was referred to in the literature review section. This data would have all the biases and inaccuracies that were discussed in that same section so we will instead estimate per-capita wealth for each group at a value that is intuitively reasonable (with guidance from the SCF taken into account). Similarly we could attempt to estimate investment distributions using that same survey, but we will instead estimate investment decision based on the assumption of increased aversion to risk being correlated with age.

TABLE 1. ASSUMED PER CAPITA WEALTH AND INVESTMENT PREFERENCES BY AGE GROUP.

	Invested Wealth	Percent Risk- Free	Percent Bonds	Percent Stocks
Pre-Investor	$500	100%	0%	0%
Risk Loving Investor	$80,000	10%	30%	60%
Risk Adverse Investor	$150,000	50%	40%	10%

Given these assumptions and the demographic data just shown, the asset distribution over the 150 year time span is summarized in the following figure:

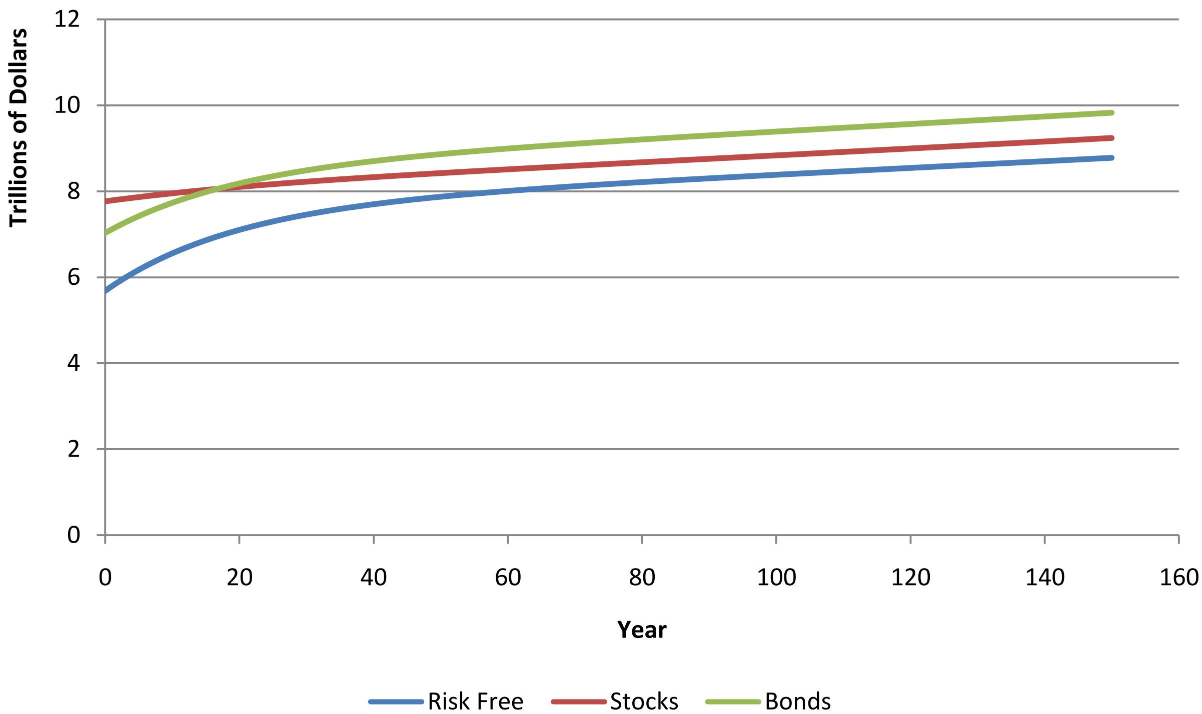

FIGURE 4: ASSET DISTRIBUTION OVER A 150 YEAR TIME SPAN.

Using the company size definitions defined in Ritter, Silber, & Udell's work and data provided by the United States Census (Bureau, Statistics about Business Size), we can approximate the initial number of businesses in each category. There are currently approximately 11 million small business, 140 thousand medium business, and 15 thousand large businesses. The α's and β's in Equation 3 are more difficult to determine. Values were chosen for this analysis based on what rendered reasonable results. Given these initial values and parameters along with the demographic and asset distribution data presented above, the distribution of company sizes over the 150 year time span is shown in the following figure:

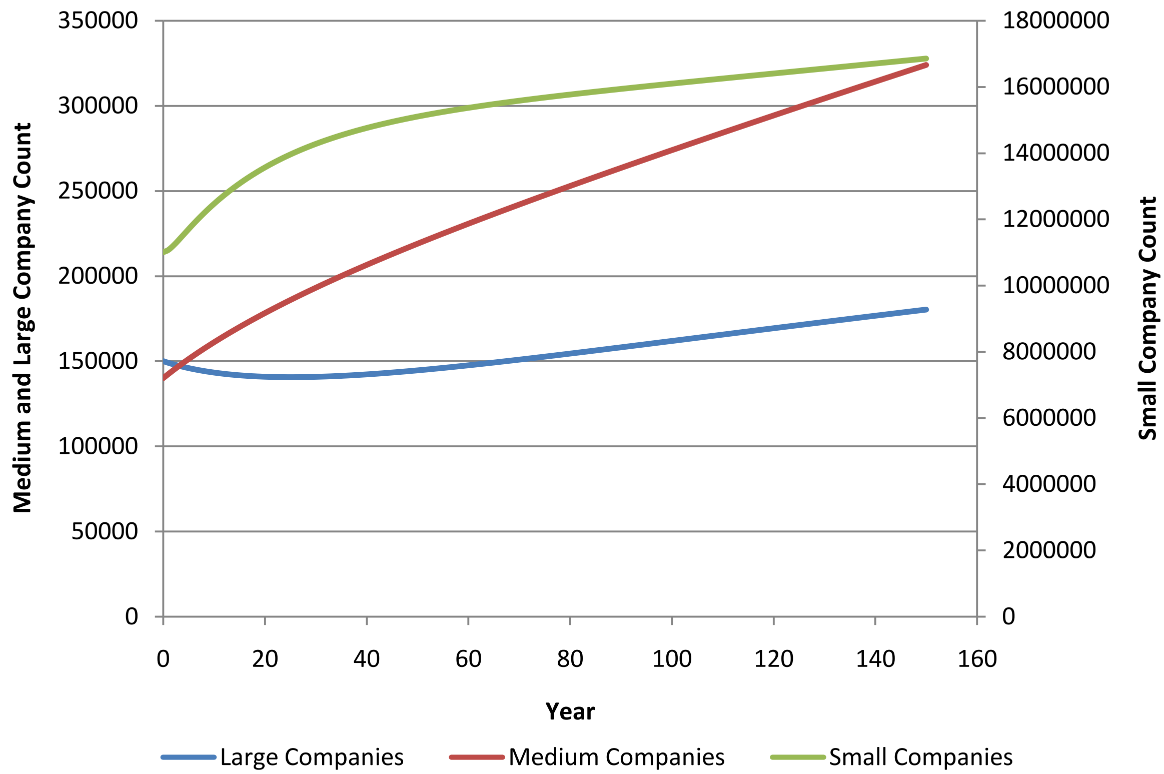

FIGURE 5: COMPANY DISTRIBUTION OVER A 150 YEAR TIME SPAN.

These calculations and figures are based on the assumption of static growth rate and static investor preferences and business conditions. We could, however, change or modify any of these variables in response to expected changes in conditions. There is not space in this simple monograph for an extensive sensitivity analysis of the various parameters, but one "what-if" scenario will be included for demonstration purposes. In this scenario we will assume that all the parameters are as specified in the above analysis except that at year 20, the fertility rate experiences a shock and drops for the rest of the simulation to 0.5 births per woman. The following three figures summarize the results of the analysis for this new scenario:

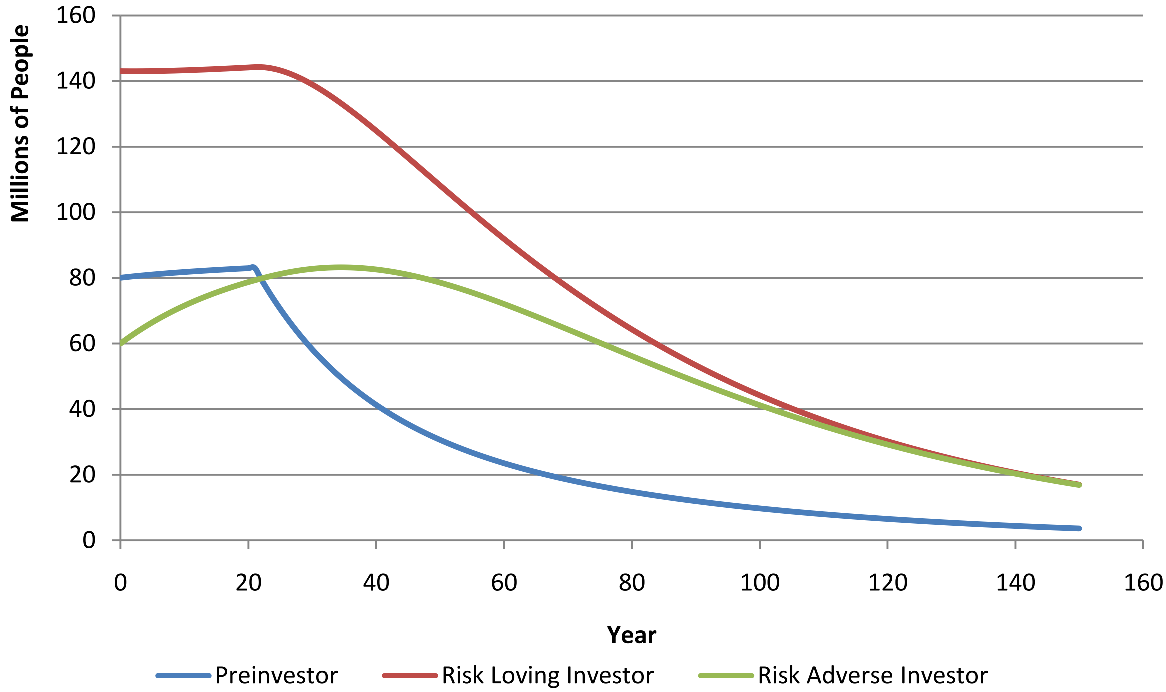

FIGURE 6: DEMOGRAPHICS GIVEN SHOCK.

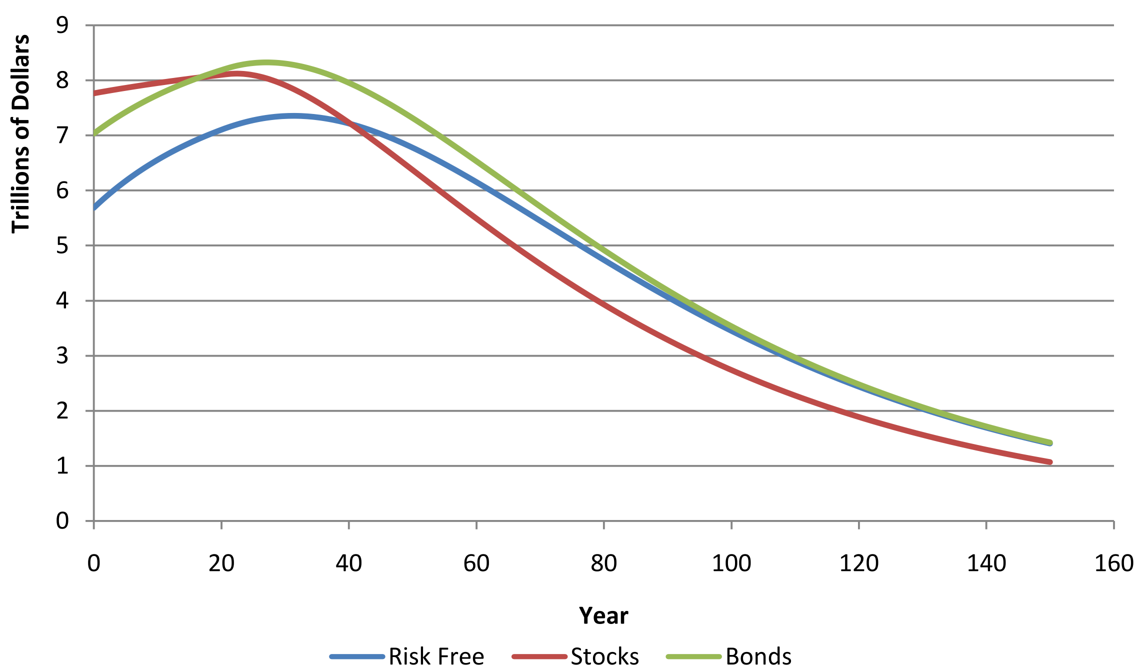

FIGURE 7: ASSET DISTRIBUTION GIVEN SHOCK.

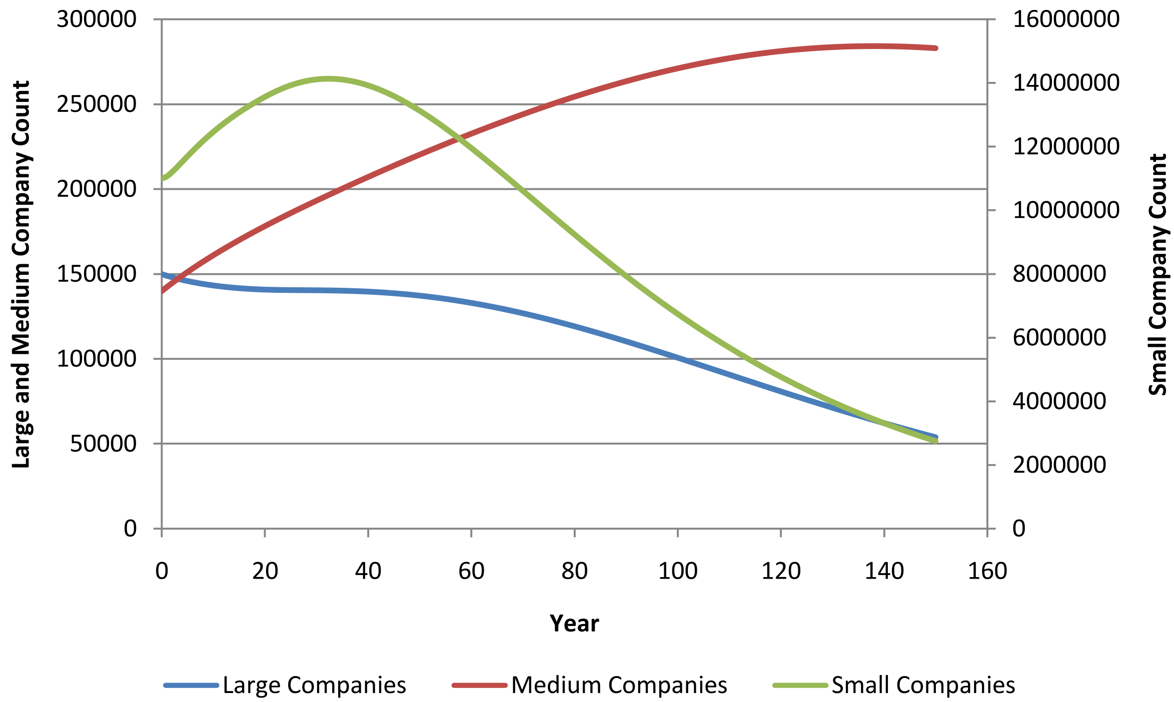

FIGURE 8: COMPANY DISTRIBUTION GIVEN SHOCK.

Figure 6 is the most easy to interpret. As the reduced birth rate comes into effect, the pre-investor population drops off sharply as would be expected. At the same time, the risk-loving investor population also starts to immediately decline but at a slower (though accelerating) rate. The risk-adverse population continues to grow for another fifteen years before declining. This simple scenario shows how it takes time for shocks to propagate through the demographic model. The changes in asset distribution are next to be discussed. All asset classes begin to fall after the shock, but stocks start falling first and most quickly. A priori, one might expect risk-free assets to fall first due to the sharp drop in pre-investors who only invest in these assets, but the small magnitude of their investments is strongly outweighed by the actions of investors in other age classes.

Company distribution is the last to be discussed and is the most interesting. Where in the first scenario small companies rapidly grew and continued to grow, in this scenario they enter a sharp decline shortly after the shock. Conversely, instead of falling, medium-sized companies actually increase during the time period under observation. The shock appears to initially have very little effect on large companies. They eventually start to fall in number, but do so at a lower rate (percentage wise) than do small companies. The ultimate result of the lowered birth rate is many fewer companies and the percentage of those companies that are small companies is much less than in the pre-shock scenario. In effect, medium-sized companies begin to dominate the market during this period of steadily shrinking population.

DISCUSSION AND CONCLUISON

This paper has presented a new model to simulate the affects of demographic changes on investment distributions between assets and their effect on the growth or contraction of companies. After conducting a literature review of existing work and models in the field, we developed the theoretical underpinnings of the DIG model. We then applied the model to a situation roughly approximating the United States. The model predicts that the number of large companies will fall over the next thirty-five years before experiencing increased growth. In the same time period both small and medium sized companies will steadily grow. These conclusions are conditional on the lack of any exogenous shocks that change the parameters fed into the model. We also applied the model to a hypothetical situation where the fertility rate dropped suddenly thereby demonstrating the versatility and power of the system dynamics approach to economic modeling. The DIG model itself is very limited as it ignores the issue of supply and demand in relation to asset prices. Nevertheless, a more detailed and powerful general equilibrium model could be developed using Systems Dynamics techniques that took these effects into account. Additionally, the possibility of birth rate being affected by company quantity and size (people being more optimistic and therefore more likely to reproduce during good economic periods) could be explored and implemented as another layer of feedback. Lastly, the possibility of large companies spinning off smaller companies should also be explored.

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